SENSITIVITY ANALYSIS OF A DYNAMIC MODEL FOR SUBMERGED ARC SILICON FURNACES

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ABSTRACT

The paper presents sensitivity analysis for a dynamic model of submerged arc silicon furnaces. The model we study, called "Simod", was developed by Elkem in cooperation with NTNU and Sintef. It is today a much used tool in Elkem for operator training and analysis of the dynamic behaviour of the silicon production process. The aim of our work is to bring the simulator tool closer to industrial practice, and construct a mechanism to adapt the model to the behaviour of a running process, online. The paper presents the results of the sensitivity analysis that has been undertaken for important parameters in the Simod model. The parameters associated with the process inputs are electric power loss, loss of fixed C and reactivity of carbon. The output signals that have been investigated are fed quartz, tapped silicon and silica. The paper also interprets the results of the sensitivity analysis in a process operation context.

1. INTRODUCTION

The aim of the work described in this paper is to use systematic sensitivity analysis to identify a limited set of model parameters for online updating of a nonlinear, dynamic model of a silicon furnace. We have identified a parameter set that has a distinct effect on the model outputs. These parameters can be estimated using the input and output data from the real process.

The process we investigate is a submerged arc silicon furnace, and the model (Simod) is a nonlinear, dynamic model developed by Elkem, NTNU and Sintef. The model is mainly used for training and for what-if simulations by process personnel, metallurgists and other specialists. Significant research has been spent in order to assure that the model represents the process as accurately as possible. There are several such furnaces in Elkem, and the model has been parameterised and configured to represent a number of these furnaces. However, the model has until now mainly been used for off-line analysis. Elkem now wants to bring the Simod model closer to the daily operation. Thus, there is a need to adapt the model to process data, online, in order to have a model that represents the current process conditions. In online adaptation of the model, slowly varying process parameters are of particular interest.

A sensitivity analysis using a candidate set of model parameters tells us about the impact of each parameter on each process output, the relative impact of the parameters and the maximal number of parameters that can be used or estimated given the outputs. The sensitivity analysis may also give important guidance in the choice of the parameter estimation method.

In section 2 we give a short description of the submerged arc silicon furnace. Section 3 gives a description of the Simod model. Section 0 discusses the choice of candidate parameters for online updating. Section 0 presents the basic mathematics behind the sensitivity analysis. The results of the parameter sensitivity analysis are given in section 0. The results of Section 0 can also be interpreted in a process operation context, and are thus of interest not only to modellers, but also to those involved in daily operation of silicon furnaces. Section 7 contains discussion and conclusion.

Proceedings: Tenth International Ferroalloys Congress; 1 – 4 February 2004
INFACON X: '*Transformation through Technology*' Cape Town, South Africa
ISBN: 0-9584663-5-1 Produced by: Document Transformation Technologies

2. THE PROCESS

Simod was originally developed as a knowledge repository for Elkem, as a basis for having a common understanding and to aid teaching the complex dynamics of the chemistry and thermodynamics of the silicon furnace. In the following section a short introduction to the basics of the silicon furnace is given before we describe the model.

2.1 Silicon furnace fundamentals

The raw materials of the silicon furnace are quartz (SiO_2) and carbon, and the product is Silicon (Si) in the form of liquid "metal" and SiO gas which oxidizes to SiO_2 to form small silicate particles called silica. The overall and highly idealized reaction stoichiometry is:

$$SiO_2 + (1+x)C = xSi + (1-x)SiO + (1+x)CO \quad (1)$$

where x is Silicon yield (i.e. tapped Silicon vs. fed Quartz rate).

The silicon furnace is a large pot in which the raw materials are fed at the top, and silicon is tapped at the bottom. Formation of silicon takes place in the lower, hottest part, called the hearth. The upper, cooler part is called the shaft. Energy is supplied through 3 carbon electrodes, and most of the electric energy is released through the electric arcs in the hearth. Here, silicon-oxide gases are also formed. These gases travel up through the feed of the furnace and react with the carbon to form silicon carbide. This is a solid that will travel downwards with the raw materials and enter the silicon producing reactions of the hearth. If too much carbon is fed to the furnace, too much silicon carbide will be produced, and there will be an undesirable build-up of silicon carbide in the hearth. This reduces silicon production. If too little carbon is fed to the furnace, a lot of silicon oxide gas will escape over the top of the furnace, and form silica fume. For more details on the silicon process and the reactions see [2]. For more information about the electrical system, see [3].

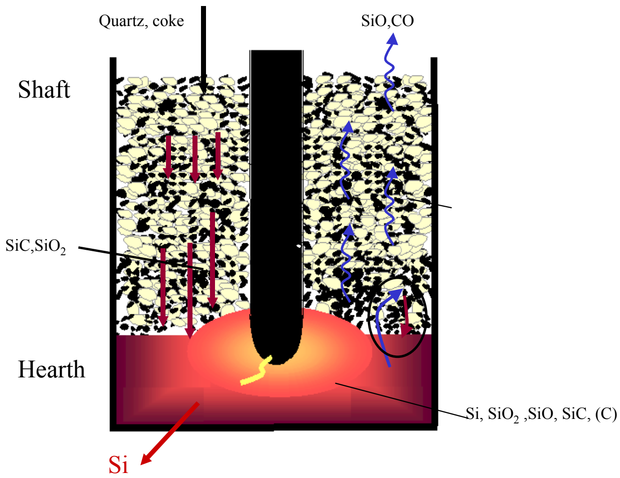

Figure 1. Mass transport in the Silicon furnace.

2.2 Measured inputs and outputs of the process

The main inputs to the process are electric energy supply, the fed quartz and carbon ratio. Quartz feeding is based on maintaining a constant and sufficiently high level of charge material in the furnace. Carbon is fed relative to quartz to optimize the process chemistry. The stoichiometric ratio of carbon vs. quartz is called fixed C [2]. Since there is normally always a loss of silicon as silica fume, the fixed C value should lie under 100% to avoid silicon carbide build-up in the hearth, ref. the overall reaction in section 2.1.

The number of online measurements is limited due to the hostile environment inside the silicon furnace. The low number of good quality measurements limits how many and which parameters can be identified online. In this work, we have considered the measurement of electric power, fixed C, fed quartz, tapped silicon and silica production to have a sufficiently good quality to be used for parameter estimation. In many processes the amount of silica produced is not measured, or not measured individually for one furnace. Therefore it is of interest to find out if this measurement affects the identifiability of the parameter set.

3. THE SIMOD MODEL

The Simod model is a nonlinear, dynamic representation of the mass balances, chemical reactions and thermodynamics of the silicon furnace. The model is one-dimensional, and no gradients are assumed in the horizontal direction. The model is based on the equations described in [2].

The main mass transportation mechanisms in the model are:

- Solid materials; carbon, quartz and silicon carbide travelling downwards.
- Liquid phase materials, melted quartz, condensed SiO (melted quartz and Si) travelling downwards.
- Gas phase, SiO and CO travelling upwards.

Once these balances have been developed, the Simod model can be written in the Differential Algebraic Equation (DAE) form:

$$\begin{aligned} \dot{x} &= f(x,y,z) \\ \dot{y} &= g(x,y,z) \\ h(x,y,z) &= 0 \\ x(t_0) &= x_0 \\ y(t_0) &= y_0 \end{aligned} \tag{2}$$

In [4] it is assumed that the dynamic states of the model can be divided into slow and fast modes, x and y. z is an auxiliary variable. The dynamics of the gas, liquid and solid phases have a large span in time constants resulting in a very stiff model.

The stiffness problem is handled using the following approach:

- The gas dynamics, i.e. the hold-up of concentration of the gas species is ignored by converting the differential equations for the fast dynamics into algebraic equations. This is done by associating y with the hold-up or concentration of gas species and setting $\dot{y}=0$ in the equations above.
- y and z will be computed prior to the integration by exploiting the structure of the model equations.
- The dominant dynamics (dynamics of solids) is integrated using a numerical integration scheme for non-stiff systems.

The furnace model is divided horizontally into compartments with homogenous process conditions within each compartment. The total model has 102 states. The solution scheme for the algebraic equations and the integration scheme for the dynamic part of the model has proven efficient and robust through approximately 4 years of use.

Extensive work has been undertaken by Elkem personnel to verify the model's behaviour vs. the dynamic process. The model can be parametrised to represent furnaces of different physical dimensions and therefore represent different plants.

The model has been programmed in C++ and has a COM-interface, and is normally run from a simulation tool, called Simon. In this study, the Simod model has been run from Matlab using a specially designed interface.

4. CHOICE OF CANDIDATE PARAMETERS

For online parameter estimation, candidate parameters are:

- Electric power loss
- Reactivity of the carbon materials
- Charge level (setpoint)
- Electric energy distribution
- Loss of fixed C
- Carbon from electrode to the hearth
- Direct fume rate (from hearth)

The total load to the furnace is measured as the electric power entering the electrodes. A fraction of this power is however lost in the furnace transformer, and some power is consumed in the electrodes themselves. The electric power loss is expressed as a loss fraction of the total power. The loss fraction is assumed to vary slowly.

The reactivity of the carbon materials is determined through laboratory analyses. In the process the carbon is composed of several types of coal, coke and wood-chips. In the model, the carbon materials are represented by only one or two types of carbon material with a specified reactivity. Because of this lumped representation of the carbon materials and uncertainties in the measurements of the reactivity of the real carbon materials, the reactivity parameter in the model is a candidate for estimation.

The feeding of the raw materials and tapping of silicon are represented with on/off controllers in the model. Altering the height setpoint for the height of the charge in the shaft affects how much silicon oxide gas escapes the furnace. Thus the height setpoint is also a candidate parameter.

The electric energy distribution in the Simod model is determined by a single parameter. This parameter specifies which fraction of energy is to enter the hearth, and which fraction is to enter the shaft. The energy fraction to the hearth is assumed to vary over time due to varying electric conductivity in the shaft and the hearth. The energy distribution can not be measured directly online, but will affect the outputs of the furnace indirectly. Thus this is also a candidate for estimation.

Loss of fixed C expresses the fact that some of the carbon fed to the furnace is burned off at the furnace top and never enters the reaction zone. This loss may vary depending on the furnace conditions. Thus, the loss of fixed C is a negative offset in the registered fixed C value.

The electrodes are consumed. Thus there is some carbon entering the hearth directly from the electrodes as well. The electrode carbon has lower reactivity than the carbon fed at the top. Electrode carbon to the hearth is therefore also a candidate parameter for online estimation.

Direct fume rate expresses the rate of silicon oxide gas that escapes the hearth directly without being in contact with the feed in the shaft. The physical interpretation of this parameter is that there is some loss of silicon oxide from the tap hole, and when channels are formed in the shaft up through the charge.

In selecting among these parameters, the following aspects are important:

- The variations in one parameter must have an effect on one or more of the model outputs that are considered.
- The variations in the parameter should give responses in the model outputs that are realistic compared to the real process.
- The parameters must be chosen so that they together form an identifiable set together with the outputs and the model.
- The chosen parameters should preferably have an intuitive physical interpretation such that their variations also are useful in the process operation.

Preliminary investigations show that loss of fixed C, electric power loss and reactivity are good candidate parameters. They all have a distinct effect on one or more of the outputs. Carbon feed directly to the hearth has not yet been studied. Thus the results reported later in this paper only consider the parameters electric power loss, reactivity and loss of fixed C.

In the online parameter estimation scheme that we are developing, the parameters mentioned above enter the model as shown in Figure 2. Above the dashed line is the real process with real inputs (fed quartz, electric power and fixed C). The text regarding the real furnace is in italics. The Δ_L and Δ_F indicate the unknown losses in the registered electric power and fixed C.

Below the dashed line is the model running on a computer. As previously mentioned, the quartz and carbon feeds have been implemented as controllers in the model. These controllers are part of the model, but are shown on the outside of the model to better illustrate the flow of signals. A future parameter estimation algorithm is indicated as a box with dashed lines. This algorithm adjusts the electric power loss, loss of fixed C and reactivity based on deviations between the real outputs from the process (fed quartz, tapped silicon and silica) and the corresponding simulated outputs from the model.

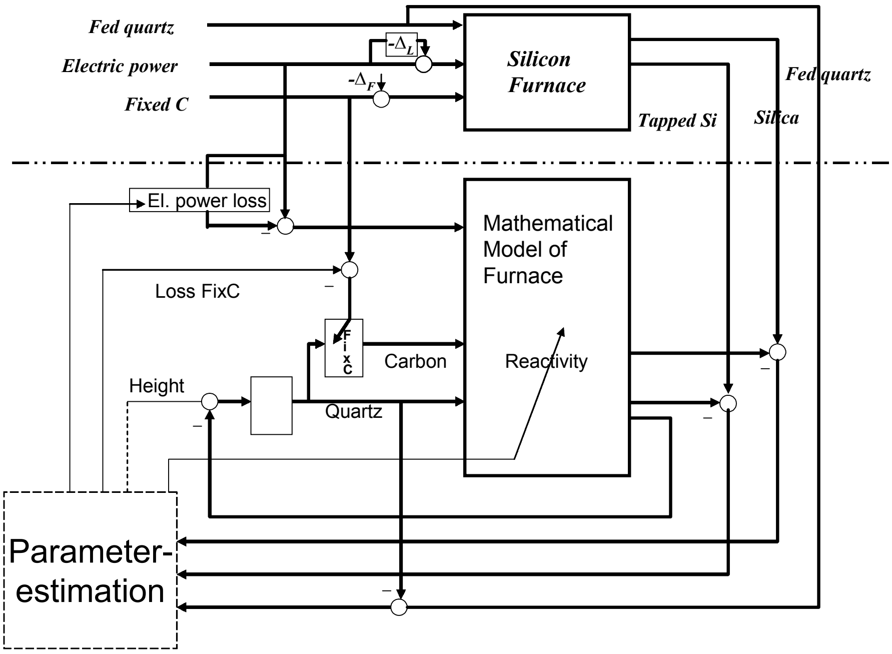

Figure 2. Overall concept for parameter estimation.

Note that we are not using the fed quartz of the process as an input to the model, but rather as an output which is compared with the amount of quartz fed to the model. From an operation perspective, fed quartz is clearly an input. However, this input is so noisy that using it gives unstable behaviour. Instead we let the model determine its own quartz level through level control, compare the real and simulated quartz feed amount, and use this deviation to tune the parameters.

5. METHODS FOR PARAMETER SENSITIVITY ANALYSIS

The final aim of the sensitivity analysis is to fit the outputs of the model to real process data through adjustment of the parameters. In our work we examine the sensitivities of the model outputs with respect to changes in the parameters values to give a basis for selecting an identifiable parameter set and to select a parameter estimation criterion and/or method.

The sensitivity matrix is given by the mathematical expression:

$$S = \frac{\partial \hat{y}(\theta)}{\partial \theta} \tag{3}$$

where $\hat{y}(\theta)$ is the output vector and θ is the parameter vector.

The sensitivity matrix has dimensions (*np x ny*), where *np* is the number of parameters and *ny* is the number of outputs. The output vector can be composed of the outputs at one time instant, or of the outputs from several time instants, stacked up to form a large matrix.
If the parameters and measurements are in a different numerical range, this may cause large differences in the sensitivities. Thus, the variables should be scaled so that:

$$\bar{y} = S_y y \qquad \text{and} \qquad \bar{\theta} = S_\theta \theta$$

where $\bar{y}$ and $\bar{\theta}$ are the scaled variables, and S_y and S_θ are symmetric scaling matrices. This gives the scaled sensitivity matrix:

$$\bar{S} = S_y \frac{\partial \hat{y}(\theta)}{\partial \theta} S_\theta \tag{4}$$

It is necessary that changes in the parameter vector affects the outputs, preferably in a unique manner. Relevant individual sensitivity measures for each parameter i are the mean square sensitivity and the mean absolute sensitivity over all the outputs, j:

$$\delta_i^{msqr} = \frac{1}{N}\sum_{i=1}^{N} \bar{s}_{ij}^2 \qquad \text{and} \qquad \delta_i^{abs} = \frac{1}{N}\sum_{j=1}^{N} \left|\bar{s}_{ij}\right| \tag{5}$$

The sensitivity matrix can also gives an indication of which methods are possible to use for estimation, or how an estimation objective should be formulated if an optimization based method is to be used. Even though no parameter estimation method has actually been chosen at this stage, a least squares criterion will most often be considered for fitting the simulator outputs to the real data containing noise. Assuming that we start with non-scaled quantities, the least squares deviation between real and simulated data is:

$$\Phi(\theta) = \frac{1}{2}(y - \hat{y}(\theta))^T W^{-1} (y - \hat{y}(\theta)) \tag{6}$$

Here y are the real outputs from the process, and W is the measurement noise covariance. It is of interest to examine the second derivative, the Hessian, of this expression since the condition number (being the ratio between the highest and lowest singular value of the matrix) of the Hessian is a measure of the curvature of the objective function close to the solution. A high condition number is an indication that a solution method for the criterion may have difficulties finding a unique parameter vector that minimizes the criterion. A high condition number may either be caused by little sensitivity to one or more of the parameters or parameter directions, by linear dependence between the columns of the sensitivity matrix or it may be due to poor scaling.

If we assume that θ is close to its optimum value, meaning $(y - \hat{y}(\theta)) \approx 0$, the approximated (unscaled) Hessian is:

$$\frac{\partial^2 \Phi}{\partial \theta^2} = S^T W^{-1} S \qquad 7)$$

If we want to scale the with respect to the parameters, and use the fact that $SS_\theta = S_y^{-1}\bar{S}$, the Hessian with respect to the scaled parameters is:

$$\frac{\partial^2 \Phi}{\partial \bar{\theta}^2} = \bar{S}^T \bar{W}^{-1} \bar{S} \tag{8}$$

with $\bar{W}^{-1} = S_y^{-1} W^{-1} S_y^{-1}$.

An alternative measure for the degree of dependence between the columns in the Hessian matrix, is the collinearity index [5]:

$$c_{lin} = \left(\sqrt{\lambda_{sm}}\right)^{-1} \tag{9}$$

where λ_{sm} is the smallest eigenvalue of (8), but with all columns of the sensitivity matrix scaled to length 1. If the collinearity index is large, this indicates that some of the parameters cause variations in the outputs that have similar directions, and there may be problems with the rank of the sensitivity matrix.

According to [5], identifiability problems are rare with condition numbers below 10, but frequent when above 100. In [6] candidate parameter subsets were first chosen from individual parameter sensitivity measures and next evaluated wrt. collinearity index. The experience in [6] was that a critical upper limit for the collinearity index lies between 5-20.

Many of our candidate parameters were ruled out due to the reasons explained above, and we ended up with a small initial set of parameters. This allowed us to consider a graphical presentation of the "stationary" values of an output plotted against two parameters. "Stationary" outputs means simulating with constant inputs and constant parameters until the model outputs reach stationary values. However, under stationary conditions, the model outputs still fluctuate around a mean due to on/off control of silicon tapping and feeding of quartz. Thus an estimated stationary output was found by averaging the output over a sufficiently high number of samples. A graphical presentation of the outputs also gives a valuable overview over the sensitivity of parameters to an output. This makes it also easier to interpret and validate the sensitivities obtained using real and not constant inputs.

6. PARAMETER SENSITIVITY ANALYSIS ON SIMOD

Section 6.1 contains a set of curves showing the model's stationary outputs as a function of electric power, fixed C and reactivity of the carbon. Constant electric power and fixed C were applied. Silicon yield is highly nonlinear with respect to fixed C [2]. In [1], the nonlinear relationship between produced silicon and fixed C is referred to as "the production curve" and was used as the most important concept in understanding the nonlinear nature of the process.

In section 6.2, we applied dynamic input data (electric power and fixed C) from one of Elkem's furnaces, and simulated with 3 different, constant parameter vectors. The parameters were checked for their individual impact on the model outputs as well as for collinearity and condition number of the scaled Hessian. In most silicon furnaces, measurements of the amount of quartz feed and tapped silicon are available. At some plants, the mass flow of silica from the furnace is also measured. The impact on the parameter sensitivities with and without the silica output has been investigated.

6.1 Stationary sensitivities

In this section, the simulations have been made as a function of electric power and fixed C. Electric power loss and loss of fixed C are subtracted offsets from these inputs as shown in Figure 2. The unscaled sensitivites of the loss parameters can be deduced from the electric power and fixed C sensitivities since they only have the opposite sign. Also, we will see from the figures that follow that the sensitivities to a loss in fixed C are highly dependent on the fixed C input value.

Simulations have been made using a time horizon of 50 times 8 hours. The first 20 samples have been avoided to ensure that initialization effects have died out.

6.1.1 Outputs as a function of electric power and fixed C

The curves that follow can be viewed in a production context or in a parameter sensitivity context. The reactivity of carbon was held constant at 0.56. We have assumed zero electric power loss and loss of fixed C, which means that following net electric power and fixed C values have been used:

- Fixed C [%] = [87 89 91 93 95 97 99 101]
- Electric power [MW] = [19.5 20.5 21.5 22.5 23.5 24.5 25.5]

Simulations have been made for all combinations of these inputs. The curves in the figures that follow, interpolate these outcomes resulting from each value of the parameters.

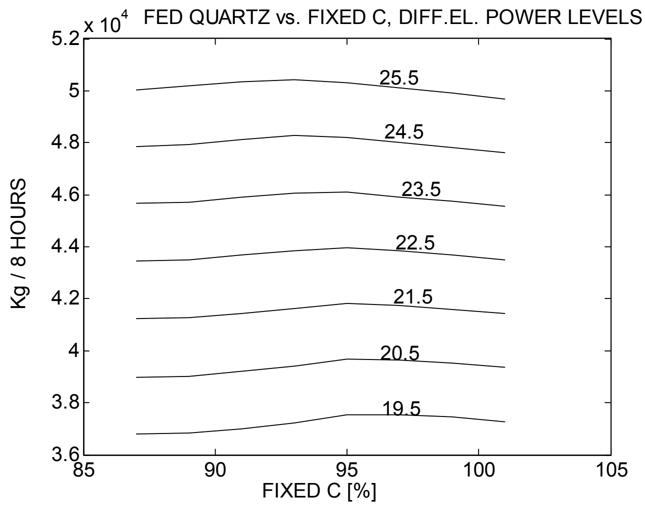

Figure 3. Fed quartz [kg/8 hours] vs. fixed C [%] for different constant electric power levels [MW].

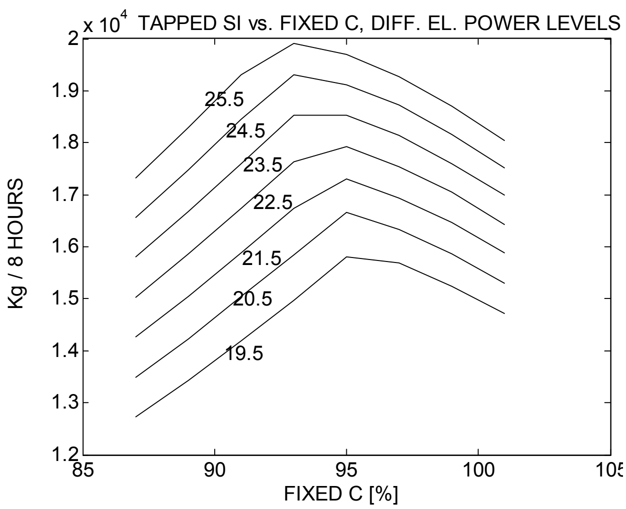

Figure 4. Tapped silicon [kg/8 hours] vs. fixed C [%] for different constant electric power levels [MW].

Figure 3 shows fed quartz and Figure 4 shows tapped silicon as a function of fixed C. Each curve corresponds to a certain constant electric power level (in MW). As we see from the figure, the quartz feed depends on the electric power level, and very little on fixed C.

We recognize the characteristic production curve for tapped silicon Figure 4 . For low values of fixed C, an increase in fixed C increases the silicon production. This only applies up to the level where the silicon carbide build-up starts limiting the silicon production. This is where the curve flattens. We use the term "critical fixed C" for the level of fixed C where the silicon production is limited by silicon carbide build-up. For even higher values of fixed C, the production level is lowered. The silicon production is also strongly dependent on the electric power level, but we also see that an increasing electric power level moves the top of the curves in the direction of lower fixed C values. An increase in the electric power increases material flow through the furnace. The hold-up time for carbon in the shaft is reduced giving the carbon less time to react with the gases. The silicon yield will therefore decrease and the furnace needs less carbon, according to the overall reaction in equation (1) section 2.1.

6.1.2 Stationary outputs as a function of reactivity and fixed C

The following curves show how the outputs depend on the reactivity of the carbon and fixed C. A constant electric power level of 22MW and a electric power loss of 10.5% has been used.

The following values have been used for the simulations:

- Fixed C [%] = [85 87 89 91 93 95 97 99 101 103]
- Reactivity of carbon materials = [0.3 0.4 0.5 0.6 0.7]

In Figure 5 we see tapped silicon as a function of fixed C for various reactivities. For low fixed C values, we see that all curves overlap. This means that the tapped silicon output is insensitive to variations in reactivity for low fixed C values. For low reactivities, the production curve reaches its maximum for a lower fixed C value than for higher reactivites. Thus, the reactivity determines the critical fixed C level. From a process operation point of view this means that high reactivity carbon gives high production of silicon. However, if the process runs with a low fixed C value, this potential does not materialize. We see that the silica production in Figure 6 is very sensitive to fixed C level. The steady state silica level is also insensitive to reactivity for low values of fixed C.

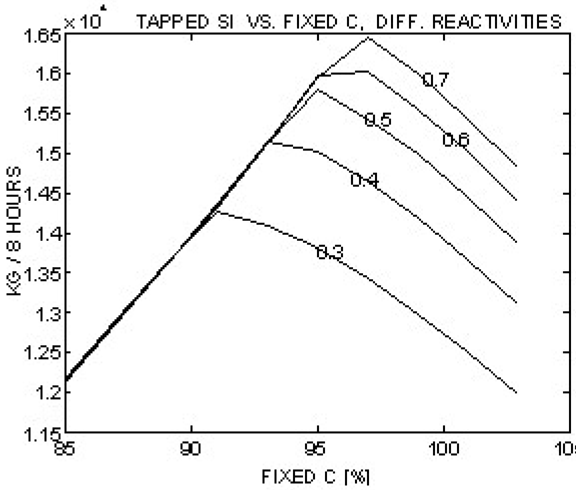

Figure 5. Tapped silicon [kg/8 hours] vs. fixed C [%] for different reactivities.

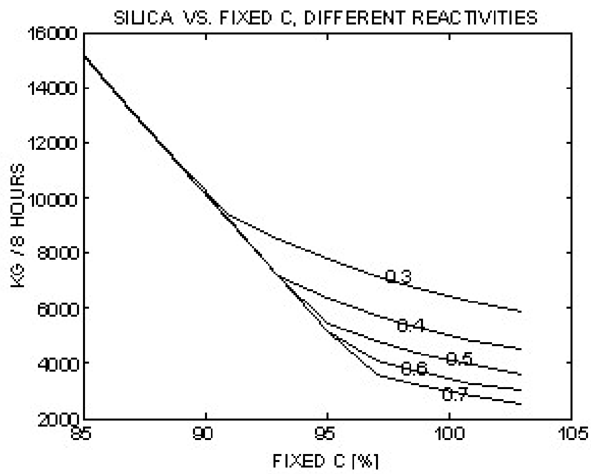

Figure 6. Silica production [kg/8 hours] vs. fixed C [%] for different reactivities.

6.2 Sensitivities using real input data

In this section we use logged 8 hour average values of fixed C and electric power inputs from a real furnace in one of Elkem's plants, see Figure 7. The model has been run using these inputs, and with 3 different parameter vectors, see Table 1. The sensitivities in the model's outputs (8 hour averaged values of) of fed quartz, tapped silicon and produced silica have been calculated using parameter perturbation and finite differences.

Table 1. Parameter vectors used.

Parameter	High fixed C	Nominal fixed C	Low fixed C
Loss electric power	10.5%	10.5%	10.5%
Reactivity	0.56	0.56	0.56
Loss fixed C	0.0%	3.0%	6.0%

The nominal loss of fixed C can be assumed to be approximately 3 %, and is referred to as "Nominal fixed C" in Table 1. Theoretical fixed C calculations based on the registered silicon yield allow us to make this calculation. If we assume 0% loss of fixed C then the fixed C values of Figure 7 would be used directly in the model. We see that fixed C lies above 95% much of the time. The parameter vector is therefore referred to as "High fixed C". The "low fixed C" parameter vector is obtained by subtracting 6% loss of fixed C. But, if we look at Figure 7, fixed C will still reach above 95% for some parts of the interval.

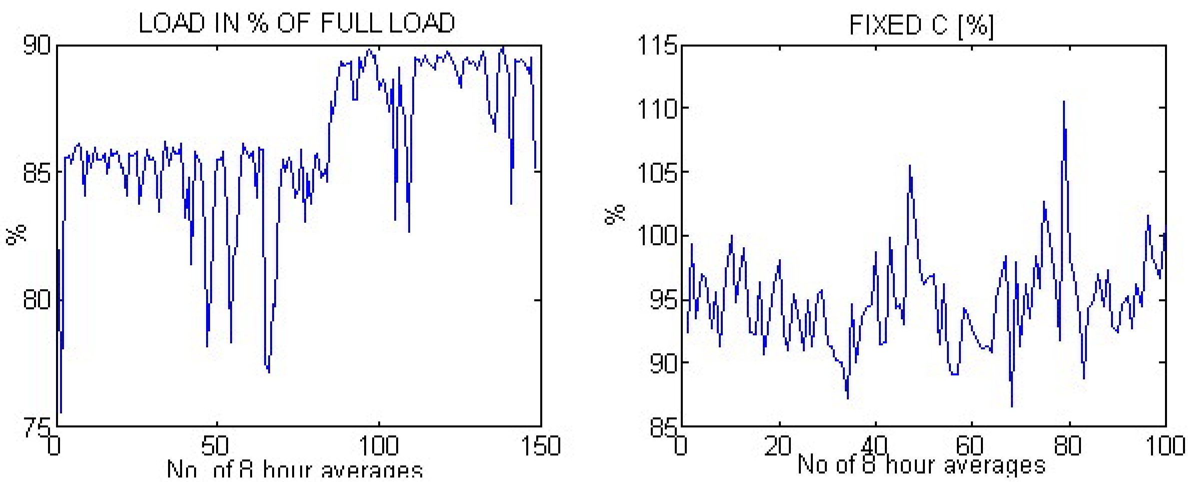

Figure 7. Electric power (in % of max. electric power) and fixed C values from an Elkem furnace used as inputs to the model.

We use a window of 100 input values, and omit the 20 first to avoid initialization effects. The scaling matrices used are $S_y = diag\left[\frac{1}{5500}, \frac{1}{3500}, \frac{1}{3000}\right]$ and $S_\theta = diag[0.04, 0.12, 2.5]$.

Each of the scaling matrices have been chosen to 25% of the variation range of the variable it scales. W has been set to unity for now.

6.2.1 Sensitivities using 3 outputs

In this section we assume that all three measurements, fed quartz, tapped silicon and the silica weight are available. The sensitivity matrix will be of dimension (240 x 3) since we include 3 measurements in 80 sampling points, and have perturbed 3 parameters. The mean squared sensitivities (3) for the 3 different parameter vectors (low, nominal and high fixed C) are given in Table 2.

The mean squared sensitivity of each parameter indicates impact of the parameter on all of the outputs. We see that the electric power loss has a very strong impact, and this impact does not vary much for different loss of fixed C values. We see that reactivity has a higher impact on the outputs for higher fixed C values than for low. The loss of fixed C parameter has the highest impact for the lowest fixed C value. These findings are all consistent with the figures of section 6.1. The mean squared sensitivity is of course scaling dependent.

Table 2. Mean squared sensitivity for each of the 3 parameters using 3 outputs.

Mean squared sensitivity	**Low fixed C**	**Nominal fixed C**	**High fixed C**
Loss electric power	0.1196	0.1294	0.1215
Reactivity	0.0382	0.0426	0.0551
Loss fixed C	0.3106	0.2830	0.1801

Table 3. Condition number and collinearity index with 3 outputs.

Parameter	**Condition no**	**Col. index**
Low fixed C	8.2983	1.1056
Nominal fixed C	6.7521	1.0704
High fixed C	3.4660	1.0939

We see from Table 3 that the condition numbers are low, and in the same range for all three parameter vectors. Still, the condition number for high fixed C is less than half the condition number of low fixed C. This may be due to the fact that the output vector is not very sensitive to reactivity at low fixed C values. The collinearity index is close to 1 in all cases. These low values for collinearity index and condition number tell us that our parameter estimation problem is well conditioned, and that we most likely would be able to find a parameter vector that minimizes for the nonlinear least squares criterion (6) in section 5.

In addition, the unscaled, mean sensitivity has been included for all 3 parameter vectors. The matrices are written in the form of a table with measurements constituting rows, and the parameters constituting the columns. As we can see from these average values, the sensitivities of fed quartz and tapped silicon to load are nearly independent of fixed C level. The sensitivities of reactivity are high for high fixed C values, and low for low fixed C values. The average sensitivity in tapped silicon to loss in fixed C is high for low and nominal fixed C values, but lower for a high fixed C. This may indicate that with this "high fixed C" value, we are near the maximum of the production curve. We also see that silica production is the most sensitive to variations in loss of fixed C. The values in the tables can be compared with the figures in 6.1 (note that the sign of the electric power loss and loss of fixed C is negative).

Table 4. Average sensitivity for low fixed C.

Low Fixed C	El. Pow. loss	Reactivity	Loss fixed C
Fed Quartz	-52745	-347.47	-127.17
Tapped Si	-18883	97.222	-420
Silica	-16771	-275.03	1058.5

Table 5. Average sensitivity for nominal fixed C.

Nominal fixed C	El. Pow. loss	Reactivity	Loss fixed C
Fed Quartz	-52829	-173.73	-144.63
Tapped Si	-19040	583.33	-370.13
Silica	-13518	-926.54	986.42

Table 6. Average sensitivity for high fixed C.

High fixed C	El. Pow. loss	Reactivity	Loss fixed C
Fed Quartz	-51161	1216.1	-48.211
Tapped Si	-17763	3154.9	-120.53
Silica	-12499	-3571.2	719.34

6.2.2 Sensitivities using 2 outputs

For many furnaces, silica production is not weighed online. Thus we want to look at the sensitivities when only fed quartz and tapped silicon measurements are available. The average, unscaled sensitivities can be extracted from the tables of the previous section, only omitting the last row corresponding to silica production. The other sensitivity measures need to be recalculated since the number of elements in the sensitivity matrix will be reduced by 1/3.

From Table 7 we see that the mean squared sensitivity for loss of fixed C is significantly reduced when the silica output is removed from the output set. The silica production level is very sensitive to fixed C, thus removing this output significantly reduces the overall impact of this loss of fixed C parameter on the outputs. The electric power loss parameter still has a large impact, and the impact of reactivity is approximately the same as when 3 measurements were used. The collinearity index is slightly increased and the condition number is reduced, but is now approximately the same for all 3 values of fixed C.

Table 7. Mean squared sensitivity for each of the 3 parameters using 2 outputs.

Mean squared sensitivity	Low fixed C	Nominal fixed C	High fixed C
El. Pow. loss	0.1529	0.1771	0.1675
Reactivity	0.0533	0.0559	0.0632
Loss fixed C	0.0587	0.0579	0.0443

Table 8. Condition number and collinearity index with 2 outputs.

Parameter	Condition no	Col. index
Low fixed C	4.3213	1.3971
Nominal fixed C	4.0669	1.2732
High fixed C	4.4146	1.1975

7. DISCUSSION AND CONCLUSION

The sensitivities of fed quartz, tapped silicon and production of silica with respect to electric power loss, reactivity and loss of fixed C have been investigated. For "low" fixed C values, the silicon production is nearly linear in fixed C. For a sufficiently high fixed C value, the silicon production is limited by the silicon carbide build-up in the hearth. In this fixed C region, the stationary production curve (tapped silicon vs. fixed C) reaches its maximum which means that the tapped silicon output is nearly insensitive to small fixed C variations. The paper uses "critical fixed C" to denote the lowest fixed C value where silicon carbide build-up occurs. Reactivity is a determining factor for critical fixed C level. An increasing reactivity increases critical fixed C. Therefore the sensitivity in tapped silicon to reactivity is high at and above critical fixed C, but low below critical fixed C.

With all three outputs included, and using the outputs from 80 sampling points, the condition number and collinearity index of the scaled Hessian have satisfactory values from an identifiability perspective. When the silica measurement is removed from the set of outputs, the sensitivity to loss of fixed C is lowered. The explanation to this is that the silica measurement is highly sensitive to variations in fixed C. The silica measurement is therefore an important measurement in determining the true fixed C level in the furnace.

Even though the condition number and collinearity index indicate that it may be possible to find a solution to a nonlinear least squares parameter estimation criterion, there is still no guarantee that the values found are correct. The fewer the parameters, the more likely it is that the estimates are biased. Therefore we basically want to estimate as many parameters as possible and still keep the condition number and collinearity index low.

The average unscaled sensitivities using the dynamic inputs in section 6.2 comply with the figures of section 6.1, only with opposite signs.

Since the investigated parameters express properties of the inputs of the process, the sensitivity results may also be interpreted in a process operation context. The qualitative influence of the parameters is summarized in Figure 8. The solid arrow goes out to the current tapped silicon/fed quartz level. The steeper this arrow, the higher the silicon yield. The small dashed arrows attached to the larger one indicate the direction of change associated with a specific parameter. The arrow attached indicates the direction in which the solid arrow will move with increasing electric power. The production level increases, increasing the length of the solid arrow, but due to some increase in silica production, the arrow will get slightly less steep, i.e. the silicon yield will go down. The little fixed C arrow attached to the solid arrow indicates that increasing fixed C increases the steepness of the arrow, and thus the silicon yield. The thin line indicates the highest possible silicon yield, which corresponds to a critical fixed C value. The dashed arrow marked r1 indicates that increased reactivity can increase the possible silicon yield of the process. However, the actual fixed C level determines whether this potential silicon yield is achieved or not.

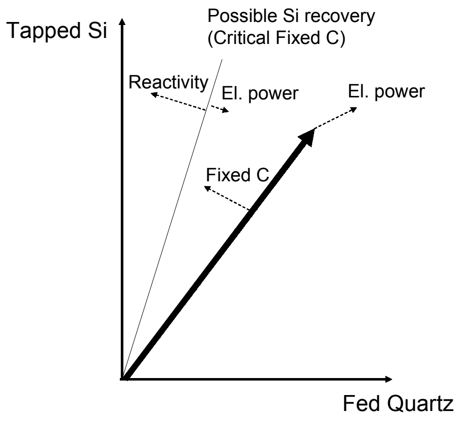

Figure 8. Qualitative effect of parameters on silicon production and quartz consumption.

8. REFERENCES

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