CONTRIBUTION MAXIMIZATION MODEL -CASE OF A FERRO ALLOYS MANUFACTURING FIRM

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ABSTRACT

Contribution is the difference between the net sales realization and cost of production. Cost of production in a Ferro alloys manufacturing depends upon the raw materials used, power consumption, and other variable expenses. In Ferro alloys production ore is the major rawmaterial which is available in different grades. Metallic contents are different at different faces, at different depths and at different locations of the mines. Contribution is a function of Sales Realization and Cost of Production. Cost of Production is function of Rawmaterial consumption, Power consumption, Rawmaterial procurement price, power tariff, and other variable expenses. Cost of material can be reduced by

1. *Using the right rawmaterial, in right quantities.*
2. *Obtaining the right blend of ore from the mines based on the quantity of ore required or to be supplied; cost of ore, and the customer satisfaction.*

Optimizing the Rawmaterial mix has a major share in maximizing the contribution as the slag volume increases with higher gang material in the charge. Use of low-grade ore, ore with higher percentage of alumina along with flux, may increase the slag volume. Since the cost of low-grade ore is less, compared to that of high-grade ore, the rawmaterial cost may decrease with increase in slag volume. As the total charge volume increases it requires more power for smelting and results in increasing in cost of power. The material mix should be selected in such a manner so as to maintain slag volume at an optimum level where the total cost will be minimum and with optimal customer mix the contribution can be maximized.

1 INTRODUCTION

Contribution is the difference between the net sales realization and cost of production. Cost of production in a ferro alloys industry depends upon the raw materials used, power consumption, and other variable expenses. The profitability of the organization depends upon two factors, i.e., Cost of production and net sales realization. In a buyer's market situation, the prices of the products are decided and fixed by the buyers on a global basis.

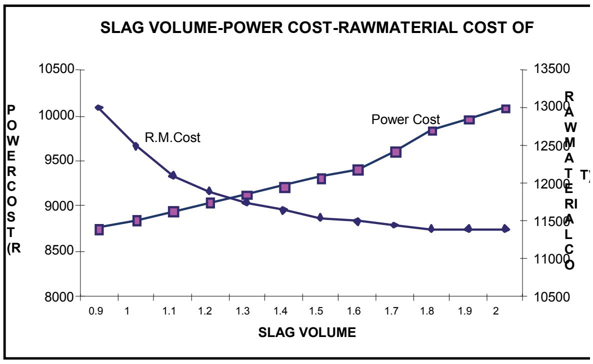
Contribution = *f* (Sales Realization, Cost of Production)

Cost of Production = *f* (Rawmaterial consumption, Power consumption, Rawmaterial procurement price, power tariff, other variable expenses)

To be competitive, the organization has to minimize the cost of production with respect to its competitors In a Ferro Alloys industry the typical variable cost base can be divided in to three major cost heads, i.e., Rawmaterial cost, Power cost, other miscellaneous variable costs. Rawmaterial and power contributes more than 85% to the total variable cost .There are about 15 to 20 varieties of raw materials that are used for manufacturing Ferro alloys. The Rawmaterial Mix can be divided in to three broad categories as 1.Basic Ore, 2.Reductants, and 3. Fluxes. Raw material (Charge Mix) is fed to the Furnace. Electrode paste goes to form soderberg electrode, which is submerged in the charge. Current is supplied through electrodes and smelting of charge takes place. Reduced metal forms alloy, and impurities join to form Slag. Ferro Alloys manufacturing is very much power intensive. Power cost is about 45% of the product cost. The power consumption is influenced by the raw material mix in the sense that higher the volume of slag higher is the power consumption

2 PRESENT PRACTICES IN THE INDUSTRY

The decision of raw material mixes to produce different alloys is mostly by heat and trial method. There is a very limited use of any optimization tools for deciding upon the material mix which leads to a variance of Rs. 2000/ per metric tonne in the cost of production between industries in addition to some seasonal variation in the cost of production within the industry. Ferro alloys production being a slag process, equal or more volume of slag is also generated along with the metal. The skill of the metallurgist to maintain a lower slag volume results in lower cost of production. Study shows a direct relationship of slag volume with raw material and power cost. Fig.1 and Fig.2 present the relationship between the slag volume with raw material and power cost and between the slag volume and total cost. Slag volume increases due to higher gang material in the charge. Use of low-grade ore, ore with higher percentage of alumina along with flux, may increase the slag volume. Since the cost of low-grade ore is less, compared to that of high-grade ore, the raw material cost may decrease with increase in slag volume.



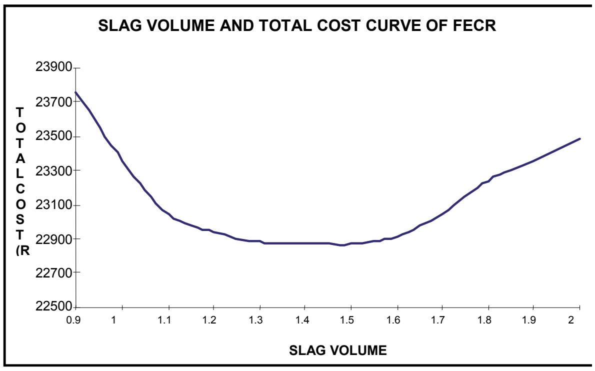


Figure 1: Slag volume-Power-Rawmaterial cost Curve

Figure 2: Slag volume & Total cost Curve

As the total charge volume increases it requires more power for smelting and results in increasing in cost of power. The material mix should be selected in such a manner so as to maintain slag volume at a level where the total cost will be minimum. Considering all the above factors, a mathematical model has been developed for the production decision of the Ferro alloys supply chain.

3 OPTIMIZATION MODEL

Contribution is a function of Sales Realization and Cost of Production. Cost of Production is function of Rawmaterial consumption, Power consumption, Raw material procurement price, power tariff, and other variable expenses. Cost of material can be reduced by using the right raw material, in right quantities. Optimizing the Raw material mix has a major share in maximizing the contribution as the slag volume increases with higher gang material in the charge. Use of low-grade ore, ore with higher percentage of alumina along with flux, may increase the slag volume. Since the cost of low-grade ore is less, compared to that of high-grade ore, the raw material cost may decrease with increase in slag volume. As the total charge volume increases it requires more power for smelting and results in increasing in cost of power. The material mix should be selected in such a manner so as to maintain slag volume at an optimum level where the total cost will be minimum. Optimum cost of production with the optimal customer mix the contribution can be maximized.

The basic objective of the model is to maximize the monthly contribution of the plant by minimizing the cost of production on the basis of raw material mix and by maximizing the net sales realization with optimal customer mix.

Xij $_{= i}$ *th Raw material for* $_j$*th Product*

XRij= $_i$ *th Reductant for* $_j$*th Product* — *(*R_1, R_2, R_3, R_4*) four types of reductant.*

XFij= $_i$ *th Flux for* $_j$*th Product* — *(*$F_1, F_2, F_3,$*) three types of flux*

XOij= $_i$ *th Ore for* $_j$*th Product* — *(*O_1, O_2, O_3, O_4*) Four types of ore*

XBij= $_i$th Briquette for $_j$th Product — *(B_1, B_2, B_3, B_4) Four types of Briquette*
XCij= $_i$th Chips for $_j$th Product — *(C_1, C_2) Two types of ore chips*

Q_j = Quantity of j th product to be produced.
V_{sj} = Slag volume from jth Product
P_j = Power requirement for the jth product
C_i = Cost of Raw material per MT.
R_i = Available Stock of i th Raw material
r = Unit rate of Power
F_c= Total Other Variable Cost (Packing, Loading, Breaking, tapping consumable, mat handling etc)
S_{ij} = Finished Product Supply Quantity to i th customer for j th Product
R_{ij} = Net sales Realization of i th customer for j th Product
j =1,2

Variable Cost (Rs/Unit) = (Cost of Raw material per MT x Consumption quantity +
Cost of Power per unit x Power consumption volume +
Total other variable cost) / Total Production Volume

Vj = Variable cost of the Product j = $(C_{ij} \times (XR_{ij} + XF_{ij} + XO_{ij} + XB_{ij} + XC_{ij}) + P_j * r + F_c) \div \sum_{j=1}^{2} Q_j$

S_{ij} = Finished Product Supply Quantity to i th customer for j th Product
R_{ij} = Net sales Realization of i th customer for j th Product

AR_{ij}=Average Net Sales Realization = $\sum_{i=1}^{n}\sum_{j=1}^{2} (S_{ij} \times R_{ij}) \div \sum_{j=1}^{2} Q_j$

Contribution (Pj)= Average Net Sales Realization (ARij)– Variable Cost of Production (Vj)
Objective Function

$$Maximise.Z = \sum_{j=1}^{2} P_j \times Q_j$$

Constraints for Product Ferro Chrome – 60-65%

$$84\% \times \sum_{i=1}^{n} Cr\%\left(XO_{i1} + XB_{i1} + XC_{i1}\right) \le 60.5\% \ge 60.1\%$$

Cr% in the alloy should be more than 60.1% and less than 60.5% in the alloy

$$98\% \times \sum_{i=1}^{n} Fe\%\left(XR_{i1} + XFi_1 + XO_{i1} + XB_{i1} + XC_{i1}\right) \le 30.5\% \ge 29\%$$

Fe% in the alloy should be more than 29% and less than 30.5% in the alloy

$$15\% \times \sum_{i=1}^{n} Si\%\left(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}\right) \le 3.5\% \ge 3.0\%$$

Si % in the alloy should be more than 3% and less than 3.5 in the alloy

$$75\% \times \sum_{i=1}^{n} Ph\%\left(XRi1 + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}\right) \le 0.035 \ge 0.025$$

Ph % in the alloy should be more than 0.025% and less than 0.03% in the alloy

$$\sum_{i=1}^{n} XC_{i1} \div \left(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}\right) \le 0.2 \ge 0.1$$

Ore chips in the in the material mix should be more than 0.10% and less than 0.20%

$$\sum_{i=1}^{n} XBi1 \div \left(XRi_1 + XFi_1 + XOi_1 + XBi_1 + XCi_1\right) \le 0.7 \ge 0.4$$

Briquette quantity in the material mix should be more than 40% and less than 70%

$$\sum_{i=1}^{n} \left(XOi_1 + XBi_1 + XCi_1\right) \le 2500 \ge 2300$$

Total Ore, Briquette and Chips quantity in the material mix should be less than 2500 kg

$$\sum_{i=1}^{n} Fc\%.in(XRi1) \geq 1.10 \times (Cr\%inAlloy \times 0.346 + Fe\%inAlloy \times 0.214 + MetalVolume \times 0.105)$$

Fixed Carbon required for smelting operation should be greater than or equal to 110% of Cr% in the Alloy x 0.346 + Fe% in the Alloy x 0.214 +Metal volume x 0.105

$$\left[\begin{array}{l} 95\% \sum_{i=1}^{n} MgO\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) \div \\ \sum_{i=1}^{n} Al_2O_3\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) \end{array}\right] \leq 1 \geq 0.7$$

The ratio of Mgo and Al_2O_3 in slag should be less than 1 and more than 0.7

$$\left[\begin{array}{l} 95\% \sum_{i=1}^{n} (Cao + MgO)\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) + \\ \sum_{i=1}^{n} Al_2O_3\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) \end{array}\right] \div 0.6 \geq 1100 \leq 1500$$

Slag volume should be more than 1100 kg and less than 1500 kg

$$\left[\begin{array}{l} 95\% \sum_{i=1}^{n} (Cao + MgO)\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) \div \\ 81\% \sum_{i=1}^{n} SiO2\%.in(XR_{i1} + XF_{i1} + XO_{i1} + XB_{i1} + XC_{i1}) \end{array}\right] \geq 1$$

Basicity of the Slag (CaO+ MgO)/ SiO_2 should be more than equal to 1

Similar set of constraints for the Product FeCr 58-60 with the constraints for FeCr 58-60 grade product can be made.

Customer Constraint.

$$\sum_{i=1}^{n} \sum_{j=1}^{2} S_{ij} \leq \sum_{j=1}^{2} Q_j$$

S_{ij} = Finished Product Supply Quantity to i th customer for j th Product
Q_j = Quantity of j th product to be produced.

$$S_{ij} \leq S_{ij}O \geq S_{ij}C$$

SijO= Finished Product Order Quantity by i th customer for j th Product
SijC= Finished Product Supply Committed Quantity to i th customer for j th Product

A typical chemical analysis of the different raw materials which is required for formulating the model is shown below in table-1

The chemical composition of Briquette (B_1) is having Cr_2O_3 content of 42.7%, FeO of 18.80%, SiO_2 of 3.2%, Al_2O_3 of 13.36%, CaO of 2.25%, MgO is 8.02% and Ph is 0.013%

Table 1: Chemical analysis of the different rawmaterial

Category	Material	Rs/MT	Cr2O3 %	FeO %	SiO2 %	Al2O3 %	CaO %	MgO %	Ph %	Cr/Fe	FC %
BRIQUETTE	B_1	1950	42.70	18.80	3.20	13.36	2.25	8.02	0.013	2.00	
	B_2	2100	46.00	15.23	3.20	13.46	3.75	7.25	0.013	2.66	
	B_3	2600	44.00	18.02	5.00	13.90	1.50	8.00	0.010	2.15	
	B_4	1600	39.00	17.17	3.20	12.23	2.25	8.00	0.013	2.00	
CHIPS	C_1	1200	46.00	16.81	3.68	17.04	0.98	9.58	0.010	2.41	
	C_2	1100	40.00	23.10	3.68	14.87	0.98	9.58	0.010	1.53	
ORE	O_1	1200	48.00	16.81	3.68	17.04	0.98	9.58	0.010	2.52	
	O_2	1100	46.00	23.10	3.68	14.87	0.98	9.58	0.010	1.76	
	O_3	5899	39.00	11.42	15.15	8.92	1.02	18.81	0.007	3.01	
	O_4	8211	50.00	12.57	6.00	8.92	1.02	18.81	0.007	3.51	
REDUCTANT	R_1	9800		1.33	8.84	3.69	0.74	0.74	0.025		86.00
	R_2	9700		1.30	8.80	3.70	0.75	0.75	0.026		86.00
	R_3	8500		0.27	2.49	0.87	0.09	0.09	0.007		89.00
	R_4	9300		0.27	2.49	0.87	0.09	0.09	0.015		89.00
FLUX	F_1	430		0.23	97.00	0.25					
	F_2	3600		0.00	5.10	2.50	1.34	44.75			
	F_3	1150		0.00	3.00	0.00	0.67	19.50			

3.1 Elemental Recovery Assumptions in the Model

It has been assumed that out of the total elemental input some percentage, of the elements will be recovered to form metal and balance will form part of slag. In the process, some percentage of the elements will be exhausted through gas and other spillage losses. The recovery assumptions are given in a tabular form in Table-2.

Table-2 Elemental Recovery Assumptions

	FC/THA	Cr	Fe	Si	P	S	Al_2O_3	CaO	MgO	C
TO ALLOY %	411	84	95	15	75	25	0	0	0	19
TO SLAG %	BTCHS/THA	14	3	81	25	75	100	95	95	
TO GAS %	1.00	3	2	2	0	0	0	5	5	

Out of the total chromium input through ore, briquette, chips etc, 84% will be recovered to metal,14% will form part of slag and about 3% will be the loss through exhaust gases and spillages.

Desired Specification of the Product

Customer requirements of the product are in terms of Chromium (Cr %), Iron (Fe), Silicon (Si) and Phosphorous (Ph) with size specification. A typical customer specification of the product is mentioned below in Table-3

Table-3 Customer specification

	FECR-60%		FECR-59%	
	Upper	Lower	Upper	Lower
Cr	60.50 %	60.10 %	59.50 %	59.10 %
Fe	30.0%	29.5%	30.0%	29.5%
Si	3.50 %	3.00 %	3.50 %	3.00 %
Phos	0.030%	0.025%	0.030%	0.025%

Considering the above data, the model can be formulated as under.

The variable cost of the Production of Ferro Chrome 60-65 grade is

$V_j = (1950\, XB_1 + 2100XB_2 + 2600XB_3 + 1600XB_4 + 1200XC_1 + 1100XC_2 + 1200XO_1$

$+1100XO_2+5899XO_3\ +8211XO_4+9800XR_1+9700XR_2+8500XR_3+9300XR_4+430XF_1$
$+3600XF_2+1150XF_4+2600P+30000000/\ (Q_1+Q_2)$

4 OUTPUT OF THE MODEL

The model was solved by using the Excel based solver optimizer. The output of the model consists of

a) **Product wise Optimum Rawmaterial Mix**
b) **Optimum Customer Mix to maximize the NSR**
c) **Summary of the output including plant profitability**

a) Product wise Rawmaterial Mix

The product wise optimum rawmaterial mix output is as shown in table- 5. The table consists of 12 columns representing the following details.

Column 1 – List of Rawmaterial.
Column 2 – Cost of rawmaterial (Rs per Metric tonne)
Column 3 –Optimum rawmaterial mix (Kg per metric tonne of finished product) for production of product-1 (FeCr-60%) and product-2 (FeCr -59%) having 60% and 59% chromium content respectively.
Column 4-12 – Chemical composition of rawmaterial consisting of Cr_2O_3, FeO, SiO_2, Al_2O_3, CaO, MgO, Ph, FC. The table also provides the chemical composition of the product, slag, and slag volume etc.

Table- 4 Product wise optimum rawmaterial mix

1	2	3	4	5	6	7	8	9	10	11	12
FECR		Product-1(60%)	Product-2(59%)	Cr2O3	FeO	SiO2	Al2O3	CaO	MgO	Ph	FC
MATERIAL	Rs/MT	Consumption (Kg/MT)		%	%	%	%	%	%	%	%
B1	1950	**22**	**0**	42.70	18.80	3.20	13.36	2.25	8.02	0.013	
B2	2100	**1097**	**726**	46.00	15.23	3.20	13.46	3.75	7.25	0.013	
B3	2600	**0**	**0**	44.00	18.02	5.00	13.90	1.50	8.00	0.010	
B4	1600	**27**	**443**	39.00	17.17	3.20	12.23	2.25	8.00	0.013	
C1	1200	**229**	**234**	46.00	16.81	3.68	17.04	0.98	9.58	0.010	
C2	1100	**1**	**0**	40.00	23.10	3.68	14.87	0.98	9.58	0.010	
O1	1200	**715**	**818**	48.00	16.81	3.68	17.04	0.98	9.58	0.010	
O2	1100	**87**	**0**	46.00	23.10	3.68	14.87	0.98	9.58	0.010	
O3	5899	**115**	**117**	45.00	11.42	15.15	8.92	1.02	18.81	0.007	
O4	8211	**0**	**0**	50.00	12.57	6.00	8.92	1.02	18.81	0.007	
R1	9800	**192**			1.33	8.84	3.69	0.74	0.74	0.025	86.00
R2	9700	**0**	**139**		1.30	8.80	3.70	0.75	0.75	0.020	89.00
R3	8500	**2**	**323**		0.27	2.49	0.87	0.09	0.09	0.007	89.00
R4	9300	**276**			0.27	2.49	0.87	0.09	0.09	0.015	89.00
F1	430	**280**	**286**		0.23	97.00	0.25				
F2	3600		**276**			5.10	2.50	1.34	44.75		
F3	1150	**404**	**79**			3.00		0.67	19.50		
TOTAL ORE (Kg/MT)		**2291**	**2337**								
Elemental input(Kg/MT)	**60%**			1064	372	400	348	58	284	0.349	412
Elemental input(Kg/MT)	**59%**			1059	380	408	357	54	351	0.316	411

Product-1(60%)

				Lower Limit	Upper Limit	Actual		Lower Limit	Upper Limit	Actual
MGO	95%	269.95	SiO2 for slag			324.9	Cr.Recovery in Alloy			615
AL2O3	100%	348.05	SiO2 for Metal		4%	75.21	Fe Recovery in Alloy			265
MGO/ AL2O3		**0.83**	Total SiO2 required			400.1	Total			880
CAO	95%	54.98	SiO2 available			399.9				
(Cao+Mgo)/ SiO2	**1.10**		Net SiO2 required			0.21	Metal Volume			**1000**
			LAMC%	0.4		0.40	Recovery			82%
SLAG VOLUME		**1160**					Cr% in Alloy	**60.1%**	**60.5%**	60.08%
Cr2o3 To Slag	12%	139	Briquete	50%	70%	50%	Slag/Metal			1.16
FEO	2%	23	Chips	10%	20%	10%				
			Lumpy ore	5%	20%	5%				
FC Required		412								

Product-2(59%)

				Lower Limit	Upper Limit	Actual		Lower Limit	Upper Limit	Actual
MGO	95%	333.31	SiO2 for slag			333.2	Cr.Recovery in Alloy			609
AL2O3	100%	356.98	SiO2 for Metal		4%	75.21	Fe Recovery in Alloy			271
MGO/ AL2O3		**0.93**	Total SiO2 required			408.5	Total			880
CAO	95%		SiO2 available			408.2				
(Cao+Mgo)/ SiO2	**1.15**		Net SiO2 required			0.21	Metal Volume			**1000**
			LAMC%	0.3		0.30	Recovery			81.92%
SLAG VOLUME		**1190**					Cr% in Alloy	**59.5%**	**60.0%**	59.5%
Cr2o3 To Slag	12%	143	Briquete	50%	70%	50%	Slag/Metal			1.190
FEO	2%	24	Chips	10%	20%	10%				
			Lumpy ore	5%	20%	5%				
FC Required		411								

b) Customer Mix to maximize the NSR
Table 5 represents the customer wise optimum quantity to maximize the NSR. The table consists of six columns representing the following details.

Column 1 – List of Products. (FeCr 60% and 59% chromium)
Column 2 – List of Customers.
Column 3 – Order quantity of each customer.(MT)
Column 4 – Minimum committed quantity to each customer.(MT)
Column 5 – NSR of each Customer. (Rs/MT)
Column 6 – Customer wise optimum quantity to maximize NSR.

Table-5 Customer Mix to Maximize NSR

Product	Customer	Odered quantity (MT)	Minimum supply quantity (MT)	Rate Rs/MT	Optimum Quantity(MT)
PRODUCT-1 FECR-60%	C1	825	660	32000	660
	C2	1500	1200	33000	1468
	C3	875	700	31500	700
	C4	425	400	31000	400
	C5	300	240	30000	240
	C6	1200	960	33000	1153
PRODUCT-2 FECR-60%	C1	890	800	30000	800
	C2	350	280	35000	350
	C3	300	240	32000	240
	C4	587	470	29500	470
	C5	400	320	28500	320
	C6	700	560	34750	700
TOTAL		**8352**	**6830**		**7500**

5 SUMMARY OF THE OUTPUT

The summary of the output is represented in the Table-6. The table provides the summary of the performance consisting of Product wise production volume where FeCr 60% production volume is 4620 MT and FeCr 59% production volume is 2880 MT. The cost of rawmaterial (from optimum material mix) is 9337 Rs/MT and 9482 Rs/MT for 60% and 59% respectively. Specific power consumption (linked with slag volume) is 3.717 and 3.745 mwh/MT respectively. Total variable cost of production of 60 and 59 grade of material is 20258 and 20470 Rs/MT respectively, Average Net sales realization and finally contribution in Rs/MT and total contribution of Rs 879 lacs for the organization has been targeted with the above product, customer and rawmaterial mix.

Table 6 Summary of the output

			Product-1 60%	Product-2 59%	Total
Production	MT	A	4620	2880	7500
R.M.Cost	Rs/MT	B	9337	9482	9393
Sp.Power	Mwh/MT	C	3.717	3.745	3.73
Power Cost	Rs/MT	D	8921	8988	8947
Other Var.Cost	Rs/MT	E	2000	2000	2000
Var. Cost	Rs/MT	F= [B+D+E]	20258	20470	20340
Realisation	Rs/MT	G	32301	31681	32063
Contribution	Rs/MT	H= G-F	12043	11210	11723
Contribution	Rs lacs	I= [H x A] / 100000	556	323	**879**

Major Advantages of the Model
The model developed can be used for daily/monthly / Annual planning process to find out the best ideal mix and can also be used to find out the best mix with the existing inventory of rawmaterial. This will surface out the impact of non-availability or stock out of some critical rawmaterial.

Any change in the customer specification can quickly be incorporated in the corresponding constraint equations from time to time

Major advantage of the model is to test the optimality with different sets of rawmaterials

The model is very user friendly. Person who knows the basic computer operation can use the model without any problem.

The model has been successfully implemented in the organization and the benefits have been realized.

Limitations of the Model

Though the model has been implemented successfully, it has got the following limitations.

The total furnace equipment are water cooled and prone to water leakage. During any water leakage from furnace components (pressure rings, rubber membrane, pipes etc.) the moisture content in the charge increases leading to disturbance in furnace operating temperature. This results in disturbance of metal recovery. In this situation some correction in the rawmaterial mix may have to be made to bring the operation under control.

On line Product changeover is regular phenomenon in Ferro alloys plant. During the change over period, which varies from 8 hours to 48 hours continuous correction in the charge burden, is made till the actual grades are achieved.

In case of long shut down of the furnace, the furnace gets cooled and there is a sequence of operations for increasing the full load of the furnace which generally varies from 8 hours to 24 hours depending upon the duration of furnace shut down. During this period the model does not fit due to many other external factors affecting the operations.